Photon tunneling in composite layers of negative- and positive-index media

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(Received 30 April 2004; published 25 October 2004)

The photon tunneling phenomena in the composite barrier of both positive index material and negative index medium (NIM) are analyzed. A negative barrier-length concept is introduced for the tunneling photon in the NIM, which facilitates the analysis of a composite barrier. In addition to transmission and reflection coefficients, some comments on the post-tunneling position and stationary-phase tunneling time are given.

DOI: 10.1103/PhysRevE.70.047603 PACS number(s): 41.20.Jb, 42.25.Bs, 78.20.Ci, 42.25.−p

A medium with both negative permittivity and negative permeability—i.e., with a negative refractive index [negative index medium (NIM) or left-handed medium (LHM)]—was introduced by Veselago as early as 1968 [1], but it did not receive much attention as it only existed in conceptual form. Recently, it was demonstrated experimentally [2] and the study of such materials has increased tremendously over the last few years.

Most research investigating the theoretical aspects of the NIM have dealt with the negative refraction phenomena at the NIM and positive index material (PIM) interface, as can be seen in the arguments since Valanju's controversial work [3–8]. Several works have been devoted to photon tunneling through a layer of the NIM and characteristics of evanescent waves in the NIM as well [9–14]. However, they mostly considered evanescent photons in a homogeneous layer composed of only the NIM. Recently, a few works have dealt with a one-dimensional layered structure constituted by a periodical repetition of PIM and NIM layers and have investigated the characteristics of photonic band-gap (PBG) structures [15–21]. In this Brief Report, the photon tunneling phenomena in a composite barrier of both the PIM and NIM will be analyzed and discussed by calculating transmission coefficients, reflection coefficients, and the post-tunneling position in a frustrated total internal reflection (FTIR) structure. The composite barrier under consideration in this Brief Report is constituted by a repetition of PIM and NIM layers, but not limited to a periodical repetition of them.

Let us consider the FTIR structure in Fig. 1, in which a photon is incident upon a media interface at an angle θ greater than the critical angle. It is well known that this two-dimensional photon tunneling problem can be converted into and analyzed as a one-dimensional electron tunneling problem by comparing the wave equation with the Schrödinger equation [22,23]. This analogy will be used in this Brief Report and the multiple low-index layers between two high-index materials will be called a "barrier" following this analogy of electron tunneling through a potential barrier. It is notable that at the NIM-PIM interface, the appropriate continuity conditions are those of Ψ and $1/\sigma(\partial\Psi/\partial z)$, where Ψ and σ denote the **E** field and the permeability (μ) of the material in case of TE polarization photons and the **H** field and the permittivity (ε) in case of TM photons. Here the discussion will be limited to a TE photon case. However, the TM photon tunneling can be dealt with quite similarly. Moreover, the multiple PIM and NIM layers comprising the composite tunneling barrier are all assumed to be impedance matched (i.e., $\varepsilon=\pm\varepsilon_2$, $\mu=\pm\mu_2$) [8], and layers of the PIM and NIM without impedance matching will not be considered.

When a PIM barrier of length d ($\varepsilon=\varepsilon_2,\mu=\mu_2$) is located between the PIM layers ($\varepsilon=\varepsilon_1,\mu=\mu_1$), the transmission amplitude (the square of its magnitude corresponds to the transmission coefficient) can be written as [22,23]

$$t_{PIM}(d)=\left[\cosh(\gamma d)-i\frac{\sigma_{12}k^2-\gamma^2/\sigma_{12}}{2k\gamma}\sinh(\gamma d)\right]^{-1}, \quad (1)$$

where $k^2=(\varepsilon_1\mu_1\omega^2/c^2)\cos^2\theta$, $\gamma^2=(\omega^2/c^2)(\varepsilon_1\mu_1\sin^2\theta-\varepsilon_2\mu_2)$, and $\sigma_{12}=|\mu_2/\mu_1|$ [24]. Here ω and c denote the angular frequency of a photon and its speed in vacuum. If the NIM barrier of the same length is located instead and impedance matched [8] with the above PIM barrier ($\varepsilon=-\varepsilon_2$ and $\mu=-\mu_2$), the transmission amplitude can be written as follows [23] using the relation $m_2/m_1=-\mu_2/\mu_1(<0)$:

$$t_{NIM}(d)=\left[\cosh(\gamma d)+i\frac{\sigma_{12}k^2-\gamma^2/\sigma_{12}}{2k\gamma}\sinh(\gamma d)\right]^{-1}=t^*_{PIM}(d). \quad (2)$$

The above relation that $m_2/m_1<0$ implies that in the NIM,

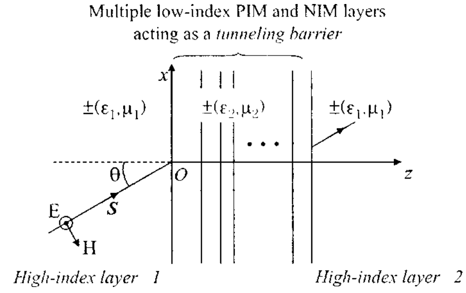

FIG. 1. TE-polarized photon tunneling through a composite barrier of multiple PIM and NIM layers. Note that in this figure, the arrows denote the flow direction of the Poynting vector.

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TABLE I. Transmission, reflection amplitudes, and effective barrier length of various configurations of layers with the NIM and PIM.

High-index layer No. 1	Tunneling barrier	High-index layer No. 2	Transmission amplitude	Reflection amplitude	Effective barrier length
PIM(ε_1,μ_1)	PIM(ε_2,μ_2)	PIM(ε_1,μ_1)	t	r	d
PIM(ε_1,μ_1)	PIM(ε_2,μ_2)	NIM$(-\varepsilon_1,-\mu_1)$	t	r	d
PIM(ε_1,μ_1)	NIM$(-\varepsilon_2,-\mu_2)$	PIM(ε_1,μ_1)	t^*	r^*	$-d$
PIM(ε_1,μ_1)	NIM$(-\varepsilon_2,-\mu_2)$	NIM$(-\varepsilon_1,-\mu_1)$	t^*	r^*	$-d$
NIM$(-\varepsilon_1,-\mu_1)$	PIM(ε_2,μ_2)	PIM(ε_1,μ_1)	t	r	d
NIM$(-\varepsilon_1,-\mu_1)$	PIM(ε_2,μ_2)	NIM$(-\varepsilon_1,-\mu_1)$	t	r	d
NIM$(-\varepsilon_1,-\mu_1)$	NIM$(-\varepsilon_2,-\mu_2)$	PIM(ε_1,μ_1)	t^*	r^*	$-d$
NIM$(-\varepsilon_1,-\mu_1)$	NIM$(-\varepsilon_2,-\mu_2)$	NIM$(-\varepsilon_1,-\mu_1)$	t^*	r^*	$-d$

the *effective* mass introduced by comparing the wave equation with the Schrödinger equation is negative. This means that if an external force is exerted on the "electron" (analogy of a photon), it moves in an antiparallel direction to that of the force. This reminds us of the fact that the direction of the wave vector and that of the Poynting vector are antiparallel in the NIM.

We can see by comparing Eqs. (1) and (2) that $t_{NIM}(d)=t_{PIM}(-d)$. That is, the NIM barrier is seen by the tunneling photon as a PIM barrier of an *effective* length of $-d$. To check further the validity of this interpretation, we calcualte the reflection coefficients for the above two cases. They can be written as

$$r_{PIM}(d)=\frac{\sigma_{12}k^2+\gamma^2/\sigma_{12}}{(\sigma_{12}k^2-\gamma^2/\sigma_{12})+2ik\gamma\coth(\gamma d)}, \tag{3}$$

$$r_{NIM}(d)=\frac{\sigma_{12}k^2+\gamma^2/\sigma_{12}}{(\sigma_{12}k^2-\gamma^2/\sigma_{12})-2ik\gamma\coth(\gamma d)} = r^*_{PIM}(d)=r_{PIM}(-d), \tag{4}$$

where we can find again that the NIM barrier is seen by the reflected photon as a PIM barrier having an effectively *negative* length $(-d)$. Therefore, we can conclude that the photon tunneling in the NIM can be dealt with in exactly the same way as in the PIM (with impedance matched), only changing the sign of barrier length or introducing a negative barrier length.

The transmission, reflection amplitudes, and effective barrier length of various configurations of layers with the NIM and PIM are shown in Table I. We need to be careful in the calculation when the plane-wave (propagating-wave) solutions are assumed in the NIM, since in the NIM the direction of wave vector ($\boldsymbol{k}$) and that of the Poynting ($\boldsymbol{S}$) vector are antiparallel. For example, in the high-index layer No. 2, the assumed solution must be $t\exp(-ikz)$ instead of $t\exp(ikz)$, and $\exp(-ikz)+r\exp(ikz)$ is assumed in the high-index layer No. 1 instead of $\exp(ikz)+r\exp(-ikz)$ (where t and r are the transmission and reflection amplitudes, respectively). We can see that the transmission, reflection amplitudes, and effective barrier length are dependent only on the sign of the refractive index of the barrier material.

The next problem is when the barrier is composed of both the NIM and PIM (having the same magnitude of refractive indices—i.e., impedance matched). After a similar analysis using the electron tunneling analogy, we can obtain inductively the following transmission and reflection amplitudes:

$$t(d_1,d_2,\ldots,d_N)=\left[\cosh(\gamma d_{eff}) - i\frac{\sigma_{12}k^2-\gamma^2/\sigma_{12}}{2k\gamma}\sinh(\gamma d_{eff})\right]^{-1}, \tag{5}$$

$$r(d_1,d_2,\ldots,d_N)=\frac{\sigma_{12}k^2+\gamma^2/\sigma_{12}}{(\sigma_{12}k^2-\gamma^2/\sigma_{12})+2ik\gamma\coth(\gamma d_{eff})}, \tag{6}$$

where the effective barrier length d_{eff} is given by $d_{eff}=\Sigma_j\,\mathrm{sgn}(n_j)d_j$ [25]. These results could be obtained independently using the above conclusion that the NIM barrier is seen by the tunneling photon as a PIM barrier having a negative length, which reconfirms the validity of our introduction of negative (tunneling) barrier length. This facilitates the analysis of the composite barrier consisting of multiple PIM and NIM layers by enabling us to convert the barrier into a PIM barrier having an appropriate effective length. We can see that it is the total sum of the "effective" length of the distributed layers of the PIM and NIM that determines the transmittance and reflectance of the overall FTIR structure. Moreover, if the total sum of the physical length of the PIM layers and that of the NIM layers are equal, the transmittance of incident waves is always one and the barrier can allow photons to tunnel through a much long distance [26].

Let us move our discussion to the post-tunneling positions. The average position where photon wave packets appear after the tunneling can be calculated using the stationary-phase approximation [23]. The shift in the x direction can be obtained by

$$\Delta x_T=\delta\ \frac{c}{|n_1|\omega\cos\theta}\frac{\partial\phi_T}{\partial\theta}\bigg|_{\omega}, \tag{7}$$

where $\phi_T=\arg(t)$, $n_1=\sqrt{\varepsilon_1\mu_1}$, and $\delta=1\ (-1)$ when photons are incident through a NIM (PIM) layer. The different values of δ in Eq. (7) originate from the different form of assumed solutions for the incident plane waves. That is, if the layer is

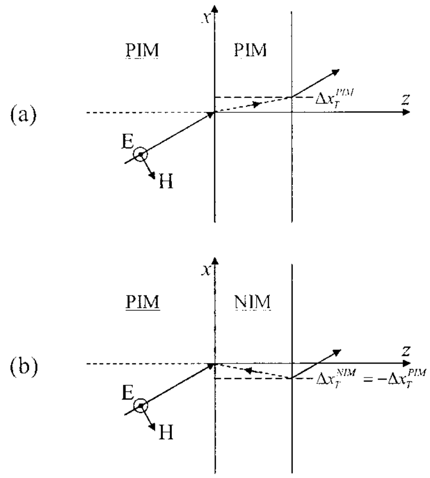

FIG. 2. Post-tunneling positions for the cases where the tunneling barrier is made of (a) a PIM layer and (b) a NIM layer. Note that in these figures, the arrows denote the directions of wave vectors.

made of a NIM, the solution becomes $\exp(-ik_x x - ikz) + r\exp(-ik_x x + ikz)$ instead of $\exp(ik_x x + ikz) + r\exp(ik_x x - ikz)$ as in the PIM, so as to make the incident energy flux direction in the NIM the same as that in the PIM. [Here $\pm k_x$ denotes the wave vector along the x direction and k_x is taken to be non-negative ($k_x = |k_x|$) for simplicity in this discussion.]

First, let us consider the case where photons are incident through a PIM layer. Using Eqs. (1), (2), and (7), we see $\phi_T^{NIM} = -\phi_T^{PIM}$ and, thus, $\Delta x_T^{NIM} = -\Delta x_T^{PIM}$. If the shift in the conventional case (no NIM layer is involved) is taken to be, say, Δx_T^0 (which is positive), we can see $\Delta x_T^{PIM} = \Delta x_T^0$ and $\Delta x_T^{NIM} = -\Delta x_T^0$. We can infer this final result on the post-tunneling position using the concept of negative barrier length, as is shown in Fig. 2(b). A barrier length that is negative can be interpreted as the tunneling direction is reversed. The "movement" of the photon through a barrier is composed of two-directional components—i.e., in x and z coordinates. Since only the tunneling direction is changed, the z-directional movement becomes reversed and the x-directional one is conserved, with which we can draw Fig. 2(b) and obtain $\Delta x_T^{NIM} = -\Delta x_T^{PIM}$.

Next, we will move to the case where photons are incident through a NIM layer. Using the results shown in Table I and Eq. (7), we can get the same relation discussed above: $\phi_T^{NIM} = -\phi_T^{PIM}$ and $\Delta x_T^{NIM} = -\Delta x_T^{PIM}$. However, due to the different value of δ in Eq. (7), we get $\Delta x_T^{PIM} = -\Delta x_T^0$ and $\Delta x_T^{NIM} = \Delta x_T^0$. These discussions are summarized in Table II and can be arranged as follows: while the post-tunneling positional shift is in the same direction as the parallel (to the interface) component of the incident energy flux when the incident and tunneling layers are of the same kind of material (i.e., PIM-PIM or NIM-NIM), it is in the opposite direction when the layers are of different kind of media (i.e., PIM-NIM or NIM-PIM).

In a composite barrier, ϕ_T is given by

TABLE II. Post-tunneling positions of various configurations of layers with the NIM and PIM.

High-index layer No. 1 (incident layer)	Tunneling barrier	Δx_T (positional shift)
PIM(ε_1, μ_1)	PIM(ε_2, μ_2)	Δx_T^0
PIM(ε_1, μ_1)	NIM($-\varepsilon_2, -\mu_2$)	$-\Delta x_T^0$
NIM($-\varepsilon_1, -\mu_1$)	PIM(ε_2, μ_2)	$-\Delta x_T^0$
NIM($-\varepsilon_1, -\mu_1$)	NIM($-\varepsilon_2, -\mu_2$)	Δx_T^0

$$\phi_T = \tan^{-1}\left[\frac{\sigma_{12}k^2 - \gamma^2/\sigma_{12}}{2k\gamma}\tanh(\gamma d_{eff})\right], \tag{8}$$

and becomes zero if the total sum of the physical length of the PIM layers and that of the NIM layers are equal. In this case, the post-tunneling position (along the x direction) is equal to the position where the incident photon hits the barrier, and the transmittance of incident waves is always 1 regardless of the physical length of the barrier. It follows that we can control the post-tunneling position of photons by changing the physical length of the PIM and NIM layers. Moreover, the post-tunneling positional shift is dependent on two factors: the kind of material comprising incident layers (through which photons are incident) and the effective length of the tunneling barrier. The shift is in the same direction as the parallel (to the interface) component of the incident energy flux (i.e., in the positive x direction in our configurations shown in Fig. 1) when the incident layer and the tunneling barrier (a tunneling barrier is said to be made of *effectively* positive or negative material depending on whether the effective length is positive or negative) are of the same kind of material (i.e., PIM-$d_{eff} > 0$ or NIM-$d_{eff} < 0$), while it is in the opposite direction when the incident layer and the tunneling barrier are of different kind of media (i.e., PIM-$d_{eff} < 0$ or NIM-$d_{eff} > 0$).

One will be tempted to calculate the tunneling time [22,23] using the same stationary-phase approximation. If its calculation is performed when, for example, photons are incident through a PIM layer and hit a PIM barrier, the tunneling time can be written as

$$\tau_T^{PIM} = \left.\frac{\partial \phi_T^{PIM}}{\partial \omega}\right|_\theta + \frac{n_1}{c}\Delta x_T^{PIM}\sin\theta. \tag{9}$$

The tunneling time with the NIM barrier is

$$\begin{aligned}\tau_T^{NIM} &= \left.\frac{\partial \phi_T^{NIM}}{\partial \omega}\right|_\theta + \frac{n_1}{c}\Delta x_T^{NIM}\sin\theta \\ &= -\left.\frac{\partial \phi_T^{PIM}}{\partial \omega}\right|_\theta - \frac{n_1}{c}\Delta x_T^{PIM}\sin\theta \\ &= -\tau_T^{PIM},\end{aligned} \tag{10}$$

and the tunneling phenomenon seems to violate causality. However, we have to be careful when dealing with the dynamic properties of the PIM-NIM interface. As was already shown in several studies [7,27] that report time-dependent simulation results, the interface acts as a strong resonance scattering center which temporarily traps the wave before reemitting it. This means that there exists a time lag between the wave hitting the interface and the medium responding with a negative index. After such a transit time, the wave reorganizes itself and starts acting as expected from the steady-state solutions. The stationary-phase approximation fails to estimate this transit time and, thus, cannot be applied to the calculation of tunneling time through the NIM barrier.

In summary, the photon tunneling phenomena through a barrier of NIM, PIM, and composite layers of both were analyzed. We find that the tunneling in a NIM can be described the same way as in a PIM by introducing a negative barrier length. This facilitates the analysis of a composite barrier of multiple layers of both PIM and NIM. We only need to calculate the effective barrier length and convert the composite barrier into one composed of only a PIM having the appropriate effective length.

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[24] If TM photons are considered, we only have to change the σ_{12} term in Eq. (1) (and in all equations and discussions in this Brief Report) into $\sigma_{12}^{TM}=|\varepsilon_2/\varepsilon_1|$. The remaining parts of the equations and all relevant discussions remain the same.
[25] To be more precise, the effective barrier length d_{eff} is given by $d_{eff}=\Sigma_j \,\mathrm{sgn}(\mu_j)d_j$ for TE photons and $d_{eff}=\Sigma_j \,\mathrm{sgn}(\varepsilon_j)d_j$ for TM photons. Therefore, it is the same for all polarization modes of photons. However, if the barrier is made of single negative materials [in which only one of the material parameters (permittivity or permeability) has negative value], the TE and TM modes of photons have different effective barrier lengths and, thus, show different features of tunneling properties such as a post-tunneling positional shift. These discussions will be reported in detail elsewhere.
[26] In this case, the overall barrier has an average index of zero. The alternate stack of PIM and NIM layers composing the tunneling barrier in this Brief Report can have both periodic and aperiodic structures. It is well known that in periodic structures (superlattices) containing both the PIM and NIM layers, there exists the PBG corresponding to an average index of zero whose features are quite different from those of conventional Bragg band gap [17,20]. This seems to be contradictory to the calculation results of zero reflection in the main text. However, the PBG corresponding to an average index of zero can exist only if there is an impedance mismatch between the PIM and NIM layers. In this Brief Report, since only impedance-matched PIM and NIM layers are considered to form the barrier, the PBG cannot show up and the zero-reflection phenomena can occur although the average index of the refraction in the (barrier) superlattice vanishes.
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